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Charged particle fluctuation as signal of the dynamics in heavy ion processes

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Abstract. We compare the dispersion of the charges in a central rapidity box according to the dual parton model with the predictions of statistical models. Significant deviations are found in heavy ion collisions at RHIC and LHC energies. Hence the charged particle fluctuations should provide a clear signal of the dynamics of heavy ion processes. They should allow one to directly measure the degree of thermalization in a quantitative way.

1 Introduction

In the analysis of the hadronic multi-particle production (for a recent review see [1]) a key observation has been the local compensation of charge [2–6]. In fixed target hadron–hadron experiments all charges in the forward region could be determined. It was therefore possible to consider the dispersion of the charge fluctuation between a forward and backward hemisphere [7–14]. In this way significant results could be obtained already in the seventies at comparatively low energies. The charge fluctuations connected to the soft hadronic part of the reactions were found to involve only a restricted rapidity range. This observation limited the applicability of statistical models to rather local fluctuations (see e.g. [15]).

Good agreement was obtained in cluster model calculations (the clusters were used to parameterize the effect of resonances). Early versions assumed neutral clusters and obtained reasonable results. The predicted Quigg–Thomas relation for the forward–backward charge fluctuation across the rapidity boundary y,

$$\langle \delta Q^2_{>y} \rangle = \langle (Q_{>y} - \langle Q_{>y} \rangle)^2 \rangle = c \cdot \mathrm{d}N_{\text{non leading charge}}/\mathrm{d}y, \tag{1}$$

was satisfied (for a review see e.g. [15]). The agreement could be improved if charged clusters with mesonic Regge exchanges [16] were allowed. As the charge structure is quite similar, the same good agreement can be expected for the dual parton model. Using the dual parton model code DPMJET [17] we explicitly checked that this is indeed the case. For p–p scattering at laboratory energies of 205 GeV good agreement with the data [18] was obtained. The Quigg–Thomas relation is satisfied in the calculation with $c = 0.70$ which compares with an experimentally preferred value of $c = 0.72$.

In heavy ion scattering, charge flow measurements should be analogously decisive. It is a central question of an unbiased analysis whether the charges are distributed just randomly or whether there is some of the dynamics left influencing the flow of quantum numbers. This is not an impractical conjecture. In heavy ion experiments the charge distribution of the particle contained in a central box with a given rapidity range $[-y_{\max.} + y_{\max.}]$ can be measured and the dispersion of this distribution

$$\langle \delta Q^2 \rangle = \langle (Q - \langle Q \rangle)^2 \rangle \tag{2}$$

can be obtained to sufficient accuracy even if some of the charges are misidentified. For sufficiently large gaps this quantity contains information about long range charge flow. In comparison to the dispersion of the forward (respectively backward) charge which has been studied at FNAL energies, the charge distribution in a central box (having two borders) can be expected to require twice the rapidity range.

It was proposed to use the quantity (1) to distinguish between particles emerging from an equilibrized quark–gluon gas or from an equilibrized hadron gas [19–21]. In a hadron gas each particle species in the box is taken essentially poissonian. In a central region at high energies where the relative size of the box is small and where the average charge flow can be ignored, one obtains a simple relation for particles like pions with charges 0 and ±1:

$$\langle \delta Q^2 \rangle = \langle N_{\text{charged}} \rangle. \tag{3}$$

The inclusion of resonances reduces the hadron gas prediction by a significant factor taken [20, 22] to be around 0.7. It is now argued in the cited papers that this relation would change in a quark–gluon gas to

$$\langle \delta Q^2 \rangle = \sum_i q_i^2 \langle N_i \rangle = 0.19 \langle N_{\text{charged}} \rangle, \tag{4}$$

where q_i are the charges of the various quark species and where again a central region is considered. The coefficient on the right was calculated [20] for a two flavor plasma in a thermodynamical consideration which predicts various quark and gluon contributions with suitable assumptions. A largely empirical final charged multiplicity

$$N_{\text{charged}} = \frac{2}{3}(N_{\text{glue}} + 1.2N_{\text{quark}} + 1.2N_{\text{antiquark}})$$

was used.

There are a number of sources of systematic errors in the above comparison between the QGP and the hadron gas. The result strongly depends on what one takes as primordial and what as secondary particles. Considering these uncertainties we follow the conclusion of Fiałkowski's papers [23] that a clear cut distinction between the hadron and the quark–gluon gas is rather unlikely. This does not eliminate the interest in the dispersion.

In the next section we discuss various possible measures to observe such fluctuations. In Sect. 3 a simple interpretation of the dispersion in terms of quark lines is outlined. An obtained proportionality suggests one to compare the dispersion to the particle density instead of the enclosed total particle multiplicity. This comparison is presented in Sect. 4 in the framework of a dual parton model Monte Carlo code (DPMJET). Modeling the statistical charge distribution by randomizing charges, the charge transfer dispersion is shown to allow for a clear distinction between string models and equilibrium approaches starting with RHIC energies. These predictions for RHIC and LHC collisions are presented in Sect. 5.

2 Various measures for charge fluctuations

For the analysis of the charge structure several quantities were discussed in the recent literature. It was proposed to look at the particles within a suitable box of size Δy and to measure just the mean standard deviation of the ratio R of positive to negative particles:

$$\langle \delta R^2 \rangle = \left\langle \left(\frac{N_+}{N_-} - \left\langle \frac{N_+}{N_-} \right\rangle \right)^2 \right\rangle, \tag{5}$$

or the quantity F:

$$\langle \delta F^2 \rangle = \left\langle \left(\frac{Q}{N_{\text{charged}}} - \left\langle \frac{Q}{N_{\text{charged}}} \right\rangle \right)^2 \right\rangle, \tag{6}$$

where $Q = N_+ - N_-$ is the charge in the box.

The motivation for choosing these ratios was to reduce the dependence of multiplicity fluctuations caused e.g. by variations in the impact parameter. In the region of interest (experimental measurements of large nuclei at high energies with a suitable centrality trigger) the charge component of the fluctuations strongly dominates and the envisioned cancellation in the density fluctuations is not important.

In this region the charge fluctuation and the proposed quantities (i.e. (3), (5) and (6)) are equivalent. They are simply connected by the following relations [20]:

$$\langle N_{\text{charged}} \rangle \langle \delta R^2 \rangle = 4 \langle N_{\text{charged}} \rangle \langle \delta F^2 \rangle = 4 \cdot \frac{\langle \delta Q^2 \rangle}{\langle N_{\text{charged}} \rangle}. \tag{7}$$

To examine the new quantities and the range where these relations hold, all three quantities were calculated in the dual parton model implementation DPMJET [17]. For the most central 5% Pb–Pb scattering at LHC energies ($s^{1/2} = 6000$ A GeV) there is indeed a perfect agreement between all three quantities as shown in Fig. 1. This agreement stays true for analogous Pb–Pb data at RHIC energies ($s^{1/2} = 200$ A GeV)[1].

Outside the region of interest of central heavy ion collisions the proposed alternatives have problems. They are not suitable for smaller Δy boxes in less dense events, as they are actually undefined ($0/0$ or ∞) if no suitable particles in the corresponding box exist.

We tried to fix the problem by ignoring undefined contributions but the result was not satisfactory as their mutual relation (7) is lost with the appearance of such terms. Specifically for the most central 5% S–S scattering at RHIC energies, the agreement is no longer good and for the minimum bias S–S scattering or for proton–proton scattering at these energies the agreement is lost.

From a string model point of view any conclusion will strongly depend on a comparison of central processes with minimum bias and proton–proton events. Our explicit Monte Carlo calculation indicates that for this purpose the more regular behaved [24] dispersion of the net charge distribution $\langle \delta Q^2 \rangle$ might be best suited.

The slightly flatter distribution for the proton–proton scattering is easy to understand. One has to realize that a glauber model does not involve just as a simple superposition. In heavy ion scattering each incoming nucleon will participate in several scattering processes. In consequence there will be more and somewhat shorter strings.

For none of the variables significant differences between rapidity and pseudo-rapidity boxes were observed. We did not investigate the influence of particles with small transverse momentum which may escape detection in the present experiments.

3 A simple relation between the quark line structure and fluctuations in the charge flow

To visualize the meaning of charge flow measurements it is helpful to introduce a general factorization hypothesis. This is not exact, but it usually holds to good accuracy. It postulates that the light flavor structure of an arbitrary hadronic amplitude can be described simply by an overall

[1] The extreme region above $\Delta y > 10$ is not relevant as it is not accessible to foreseeable experiments. Obviously, in the limit where the box extends over the total kinematic range, where $n_+ - n_- \rightarrow Z_1 + Z_2$ is fixed, the dispersions $\langle \delta R^2 \rangle$ and $\langle \delta F^2 \rangle$ are dominated by pure multiplicity fluctuations

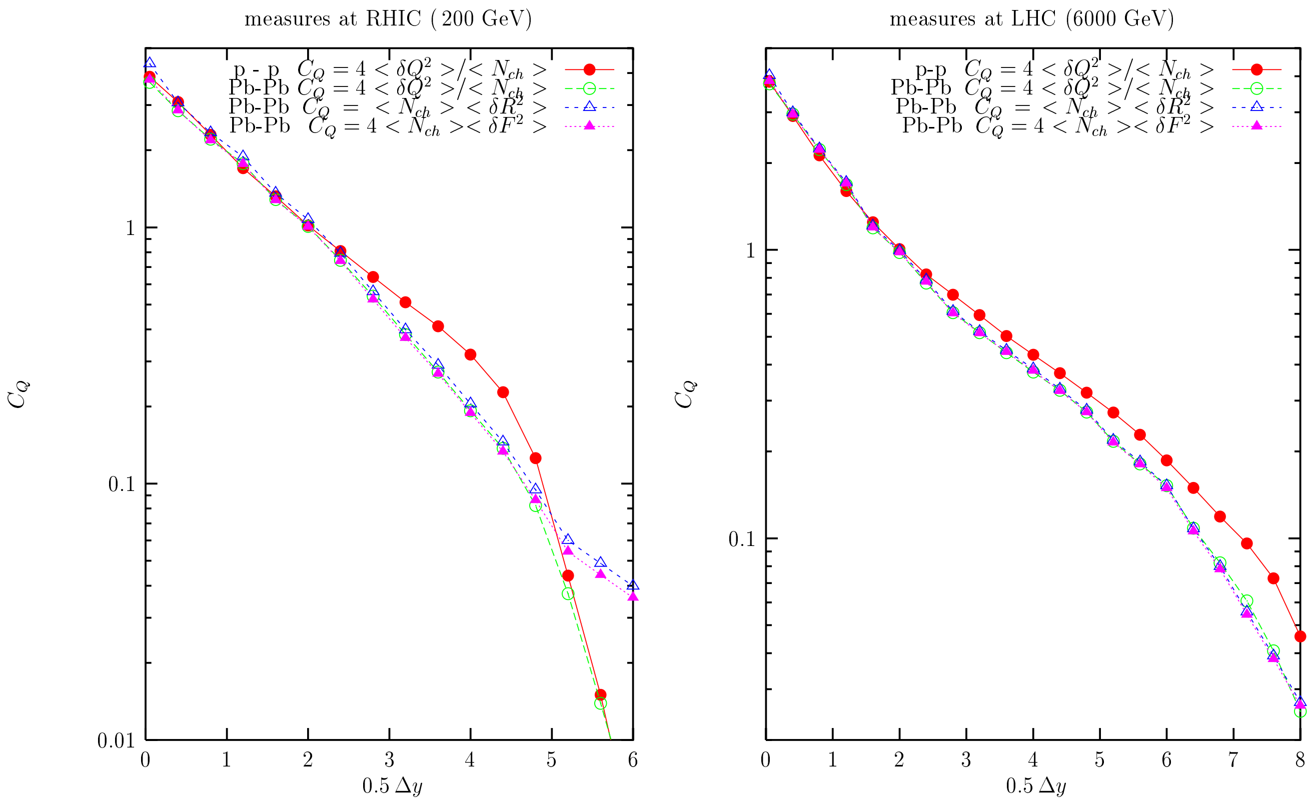

Fig. 1. Charge fluctuations for the most central 5% Pb–Pb scattering at RHIC energies $s^{1/2} = 200$ A GeV and at LHC energies ($s^{1/2} = 6000$ A GeV). Also shown are corresponding data for p–p scattering

factor, in which the contribution from individual quark lines factorize[2].

The hypothesis can be used to obtain the following generalization of the Quigg–Thomas relation [28,25,16,29]. It states that the correlation of the charges $Q(y_1)$ and $Q(y_2)$, which are exchanged during the scattering process across two kinematic boundaries, is just

$$\langle \{Q(y_1) - \langle Q(y_1) \rangle\} \{Q(y_2) - \langle Q(y_2) \rangle\} \rangle = n_{\text{common lines}} \langle (q - \langle q \rangle)^2 \rangle . \tag{8}$$

where $n_{\text{common lines}}$ counts the number of quark lines intersecting both borders and q is the charge of the quark on such a line. Depending on the flavor distribution average values $\langle (q - \langle q \rangle)^2 = 0.22 \cdots 0.25$ are obtained.

Most observables of charge fluctuations can be expressed with this basic correlation. Our fluctuation of the charges within a $[-y_{\text{max.}}, +y_{\text{max.}}]$ box contains a combination of three such correlations. Using (8) for each contribution the dispersion of the charges in a box subtracts to

$$\langle \delta Q[\text{box}]^2 \rangle = n_{\text{lines entering box}} < (q - \langle q \rangle)^2 , \tag{9}$$

where $n_{\text{lines entering box}}$ is the number of quark lines entering the box.

[2] The hypothesis is based on the exchange degeneracy of octet and singlet Regge trajectories effectively changing the $SU(N_{\text{effective}})$ flavor symmetry to an $U(N_{\text{effective}})$ symmetry in which this relation is exactly valid. Corrections to the hypothesis originate in the special behavior of the masses of the lowest lying mesons of the trajectories, which is especially significant in the pseudo-scalar sector, i.e. between the π_0 and the η meson. This introduces an anticorrelation between flavors on neighboring quarks which can be ignored in considerations concerning long range charge transfers.
If a higher accuracy is desired the hypothesis can be restricted to primary particles which are less sensitive to these masses [25,16]. The "secondary" charges (produced in pairs during the decay of large primordial particles) have then to be considered extra using a Quigg–Thomas relation [26–28] $\langle \delta Q^2(y) \rangle = \sigma(1/2)\rho_{\text{charged secondary}}(y)$ where $\sigma \approx 1$

4 Calculation of the dispersion of the charge distribution within a box

Let us consider the prediction of a thermodynamic model in more detail. In the thermodynamic limit with an infinite reservoir outside and a finite number of quarks inside, all quark lines will connect to the outside as shown in Fig. 2. The dispersion of the charge transfer is therefore proportional to the total number of particles inside. In the "hadron gas" all particles contain two independent quarks each contributing with roughly 1/4 yielding the estimate of (2). For the "quark–gluon gas" only one quark or gluon of each hadron originates in a non-local process. The other partons needed for the hadronization are assumed to be

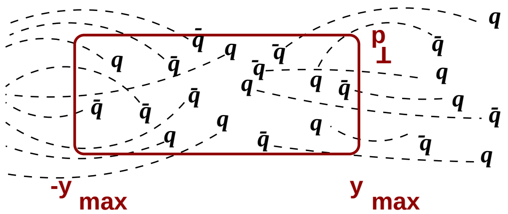

Fig. 2. Quark lines entering the box in the thermodynamic limit

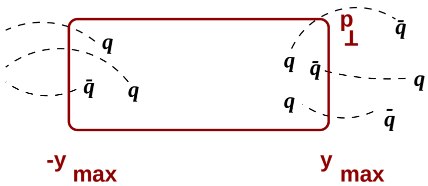

Fig. 3. Quark lines entering the box with local compensation of charge

short range so that for a box of a certain size their contribution can possibly be ignored. In this way the charge transfer is drastically reduced. Obviously there are several refinements to this simple picture.

Let us consider the limit of a tiny box. Looking only at the first order in Δy one trivially obtains

$$\langle \delta Q^2 \rangle / \langle N_{\text{charged}} \rangle = 1, \tag{10}$$

which corresponds to the hadron gas value.

If the box size increases to one or two units of rapidity on each side this ratio will typically decrease, as most models contain a short range component in the charge fluctuations. One particular short range fluctuation might be caused by the hadronization of partons of the quark gas discussed above. The decreasing is however not very distinctive. Common to many models are secondary interactions which involve decay processes and comover interactions. In hadron–hadron scattering processes such short range correlations are known to play a significant role and there is no reason not to expect such correlations for the heavy ion case.

After a box size passed the short range the decisive region starts. In all global statistical models [19,21,20] the ratio will have to reach now a flat value. Only if the box involves a significant part of the total rapidity, charge conservation will force the ratio to drop by a correction factor

$$\text{factor} = \left(\int_{y_{\text{max.}}}^{Y_{\text{kin.max.}}} \rho_{\text{charge}}^{\text{new}} \mathrm{d}y \right) \Big/ \left(\int_{0}^{Y_{\text{kin.max.}}} \rho_{\text{charge}}^{\text{new}} \mathrm{d}y \right)$$
$$\propto 1 - y_{\text{max.}} / Y_{\text{kin.max.}}. \tag{11}$$

This is different in string models. The model calculations (Fig. 1) with a rapid fall off indicate a manifestly different behavior. It is a direct consequence of the local compensation of charge contained in string models. The effect is illustrated in Fig. 3 in which only quark lines are shown which intersect the boundary and contribute to the charge flow. Now local compensation of charge allows only

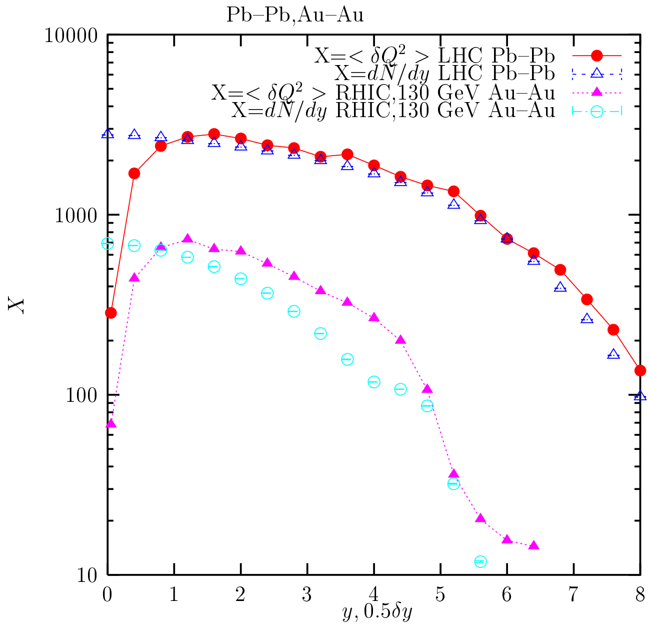

Fig. 4. Comparison of the dispersion of the charge distribution with the density on the boundary of the considered box for central gold–gold respectively lead–lead scattering at RHIC and LHC energies

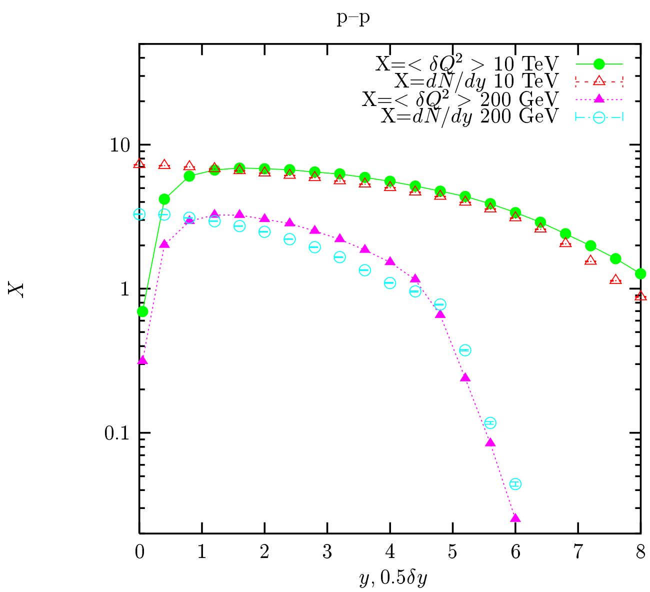

Fig. 5. Comparison of the dispersion of the charge distribution with the density on the boundary of the considered box for proton–proton scattering at RHIC and LHC energies

for a contribution of lines originating around the boundaries. If the distance is larger than the range of charge compensation the dispersion will no longer increase with the box size. The total contribution will now be just proportional to the density of the particles at the boundaries:

$$\langle \delta Q^2 \rangle \propto \rho_{\text{charged}}(y_{\text{max.}}). \tag{12}$$

It just counts the number of strings.

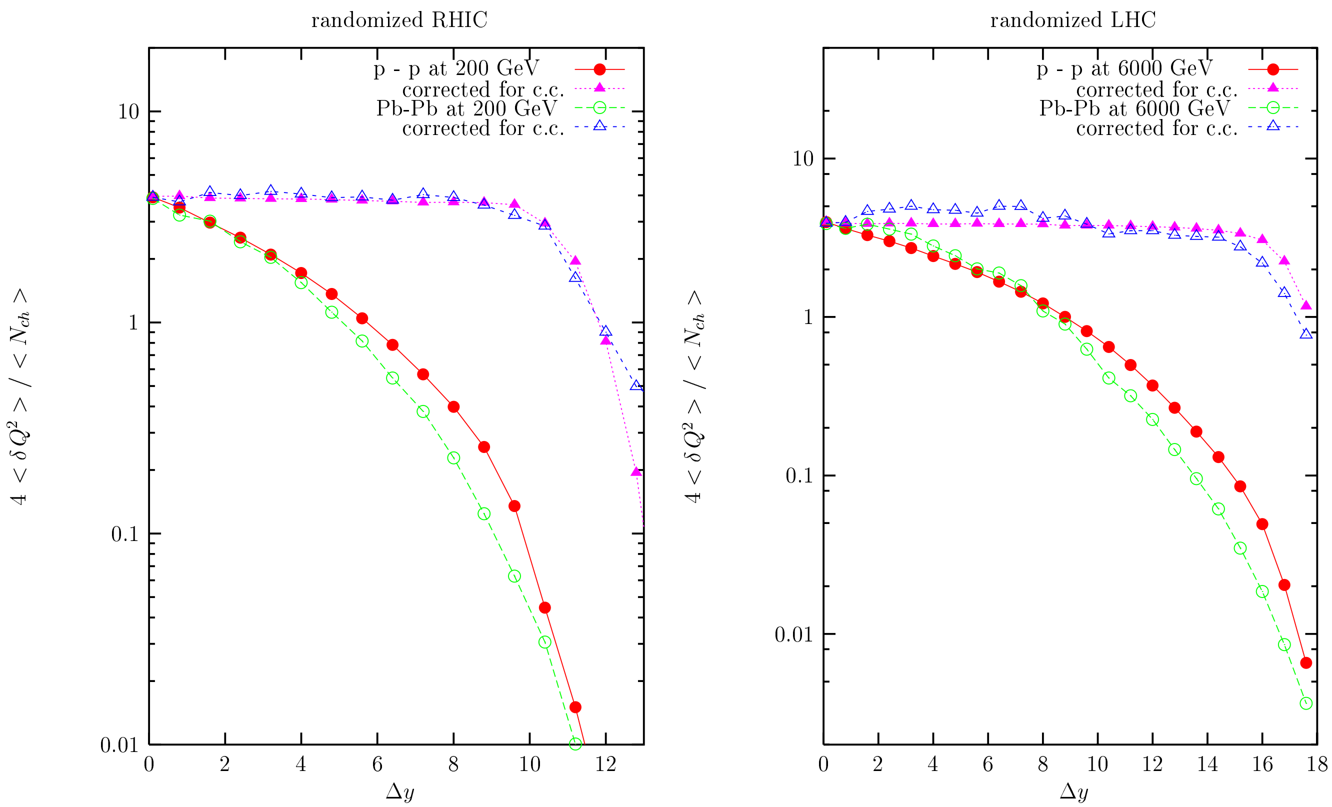

Fig. 6. Charge fluctuations with a posteriori randomized charges for *p*–*p* scattering and the most central 5% in Pb–Pb scattering at RHIC energies ($s^{1/2} = 200$ A GeV) and at LHC energies ($s^{1/2} = 6000$ A GeV). The results are also shown with a correction factor to account for the overall charge conservation

This resulting scaling, which is indeed very similar to the relation (1), which was already found in the seventies, is illustrated in a comparison between both quantities in (12) and shown in Fig. 4 for RHIC and LHC energies. The agreement is comparable to the proton–proton case shown in Fig. 5. The proportionality is expected to hold for a gap with roughly $(1/2)\delta y > 1$. For smaller boxes some of the quark lines seen in the density do not contribute as they intersect both boundaries. For large rapidity sizes there is a minor increase from the leading charge flow $Q_{\rm L}$ originating in the incoming particles. In a more careful consideration [16] one can subtract this contribution:

$$\langle \delta Q^2 \rangle_{\text{leading charge migration}} = \langle Q_{\rm L} \rangle (1 - \langle Q_{\rm L} \rangle), \qquad (13)$$

and concentrate truly on central fluctuations.

The prediction for the proportionality factor for the case of mere short range fluctuations would be roughly a factor one (see footnote 1). In string models primordial particles are responsible for a longer range charge transfer coming from the contributions of the quark respectively diquark fragmentation chains. Taking everything together one obtains

$$\langle \delta Q^2 \rangle = \sum_{\text{left+right}} \left\{ n_{\text{strings}} \cdot 2 \langle (q - \langle q \rangle)^2 \rangle + \sigma \frac{1}{2} \rho_{\text{charged secondary}}(y) \right\} \qquad (14)$$

where $n_{\text{strings}} = \rho_{\text{charged primary}} / \rho_{\text{single string}}$ is the number of strings. The width of the local fluctuations σ is roughly unity.

5 Expectations for RHIC and LHC collisions

In a recent publication Bleicher, Jeon, Koch [22] pointed out that the overall charge conservation cannot be ignored at SPS energies. They showed that their string model prediction[3] coincides with the expectation of a statistical model of hadrons and that the considered measure is therefore not sufficiently decisive in the considered energy range. Our string model DPMJET supports this conclusion for the SPS energy range as it obtains the same central-box charge fluctuations. While forward–backward hemisphere charge fluctuations were meaningful in the FNAL-SPS energy region, the fluctuations of charges into a central box contain two borders and require a correspondingly doubled rapidity range.

It was argued [22] that the experimental results should be "purified" to account for charge conservation. In our opinion a sufficiently reliable estimate of this factor is not

[3] In the energy range above $s^{1/2} = 5$ GeV the UrQMD code used by them is stated [30] to be dominated by string fragmentation

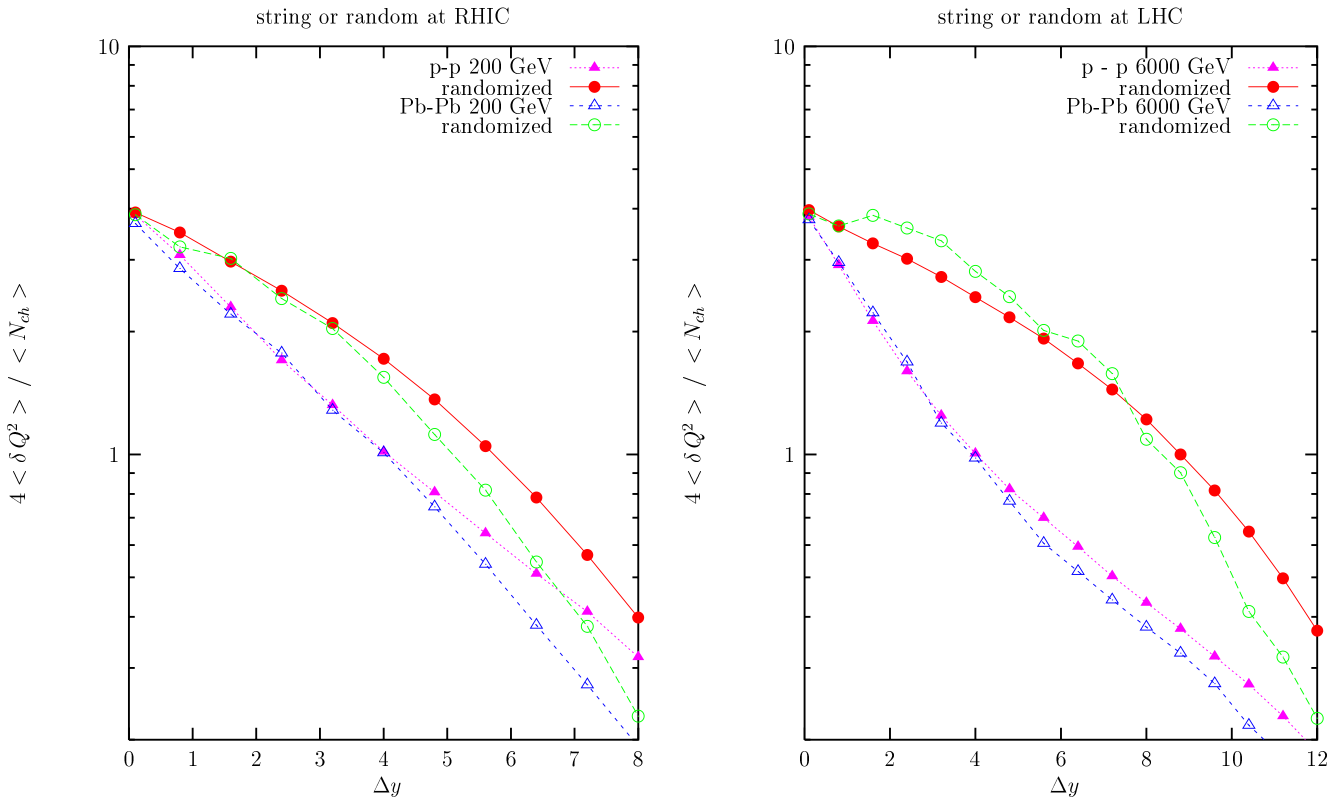

Fig. 7. Comparison of the charge fluctuations obtained in a string model DPMJET with a model using a posteriori randomized charges for p–p scattering and the most central 5% in Pb–Pb scattering at RHIC energies ($s^{1/2} = 200\,\mathrm{A\,GeV}$) and at LHC energies ($s^{1/2} = 6000\,\mathrm{A\,GeV}$)

available and the implementation of the charge conservation has to remain on the model side. The estimate of Bleicher, Jeon, Koch is based on (11). For $\langle \delta R^2 \rangle$ and $\langle \delta F^2 \rangle$ the corresponding relation holds only to first order, which seems at least on the formal side not sufficient. Even for $\langle \delta Q^2 \rangle$ it should be taken with care. One can easily underestimate the effect of charge conservation, as even in statistical models [31] not all charged particles might be fully mixed in. The leading particle often exhibit a special behavior.

To obtain an estimate in a reference model with statistical fluctuations we a posteriori randomized charges in final states obtained with DPMJET. A similar procedure to create a reference sample could be directly applied to the experimental data. To conserve energy and momentum absolutely accurately it was done in our calculation separately for pions, kaons and nucleons. The result is shown in Fig. 6 for RHIC and LHC energies for proton–proton and central lead–lead collisions. To check consistency we employed the proposed correction factor

$$1 - \int_0^{y_{\mathrm{max.}}} \rho_{\mathrm{charge}} \mathrm{d}y \Big/ \int_0^{Y_{\mathrm{kin.max.}}} \rho_{\mathrm{charge}} \mathrm{d}y$$

and indeed obtained the flat distribution with the expected “hadron gas” value.

Taking the DPMJET string model and the randomized “hadron gas” version as extreme cases we can investigate the decisive power of the measure. As shown in Fig. 7 we find that there is a measurable distinction at RHIC energies and a sizeable one at LHC.

The similarity of p–p and Pb–Pb scattering is not surprising. The distinction between both cases is expected from the difference in collective effects. The data for p–p scattering are known to follow the string models, while interaction of comovers, or medium range or complete equilibration will move the curve upward to a more statistical situation. These effects are presently outside of the model. A measured charge correlation between both extremes will directly reflect the underlying new physics.

A similar result is obtained when the dependence on the centrality is studied. Without collective effects no such dependence is expected and found in the model calculation as can be seen in Fig. 8 (b is the impact parameter). It should be stressed that this experimentally measurable centrality dependence allows one to directly observe collective effects without reference to model calculations and underlying concepts.

6 Conclusion

In this paper we demonstrated that the dispersion of the charge distribution in a central box of varying size is an extremely powerful measure. Within the string model calculation the dispersion seen in relation to the spectra

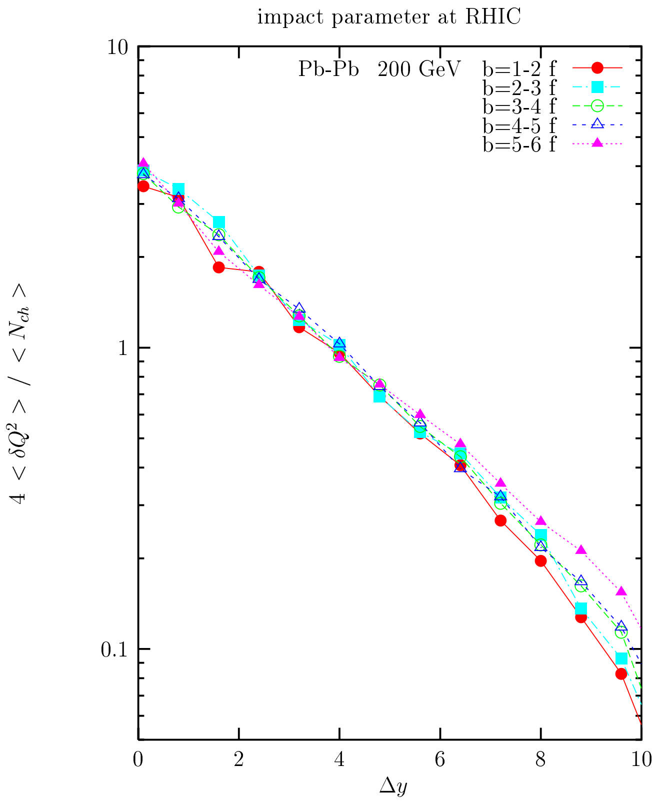

Fig. 8. The b dependence of the charge fluctuations obtained in the string model DPMJET for Pb–Pb scattering at RHIC energies ($s^{1/2} = 200\,\mathrm{A\,GeV}$)

shows no significant difference between simple proton–proton scattering and central lead–lead scattering even though both quantities change roughly by a factor of 400.

The dispersion allows one to clearly distinguish between conventional string models and hadronic thermal models for a rapidity range available at RHIC energies. In many models the truth is expected to lie somewhere in between. It is a quite reasonable hope that the situation can be positioned in a quantitative way.

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